QUIZ 3 SOLUTIONS: LESSON 2
AUGUST 28, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Evaluate $\displaystyle\int_6^{11} \frac{1}{2\sqrt{x-2}}\,dx$

Solution: Take $u = x - 2$, then $du = dx$ and

$$u(6) = 6 - 2 = 4 \text{ and } u(11) = 11 - 2 = 9.$$

Thus,

$$\begin{aligned}
\int_6^{11} \frac{1}{2\sqrt{x-2}}\,dx &= \int_{u(6)}^{u(11)} \frac{1}{2\sqrt{u}}\,du \\
&= \int_4^9 \frac{1}{2\sqrt{u}}\,du \\
&= \int_4^9 \frac{1}{2}u^{-1/2}\,du \\
&= \frac{1}{2}\left(\frac{1}{-\frac{1}{2}+1}\right)u^{-1/2+1}\Big|_4^9 \\
&= \frac{1}{2}\left(\frac{1}{\frac{1}{2}}\right)u^{1/2}\Big|_4^9 \\
&= \frac{1}{2}(2)\sqrt{u}\Big|_4^9 \\
&= \sqrt{u}\Big|_4^9 \\
&= \sqrt{9} - \sqrt{4} \\
&= 3 - 2 = \boxed{1}.
\end{aligned}$$

2. [4 pts] Evaluate $\displaystyle\int \frac{x}{2\sqrt{x-1}}\,dx$

Solution: Take $u = x - 1$, then $du = dx$ and $x = u + 1$. Thus

$$\int \frac{x}{2\sqrt{x-1}}\,dx = \int \frac{u+1}{2\sqrt{u}}\,du$$

$$
\begin{aligned}
&= \int \left(\frac{u}{2\sqrt{u}} + \frac{1}{2\sqrt{u}} \right) du \\
&= \int \frac{1}{2}\sqrt{u}\, du + \int \frac{1}{2\sqrt{u}}\, du \\
&= \frac{1}{2}\left(\frac{2}{3}\right) u^{3/2} + u^{1/2} + C \\
&= \frac{1}{3}u^{3/2} + u^{1/2} + C \\
&= \boxed{\frac{1}{3}(x-1)^{3/2} + (x-1)^{1/2} + C}
\end{aligned}
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3. [2 pts] Find the average value of the function $f(x) = 9x^2 + 3$ over the interval $2 \leq x \leq 5$.

   Solution: The average value is given by

$$\frac{1}{5-2}\int_2^5 (9x^2+3)\, dx.$$

   So,

$$
\begin{aligned}
\frac{1}{5-2}\int_2^5 (9x^2+3)\, dx &= \frac{1}{3}\int_2^5 (9x^2+3)\, dx \\
&= \frac{1}{3}(3x^3+3x)\Big|_2^5 \\
&= x^3 + x\Big|_2^5 \\
&= (5)^3 + 5 - (2^3+2) \\
&= 125 + 5 - (8+2) \\
&= 130 - 10 = \boxed{120}
\end{aligned}
$$